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# Synthesis of Kinematic Schemes for Surface Machining on Metal-Cutting Machines

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**Abstract:** This study presents a systematic approach to the synthesis of kinematic schemes for surface machining on metal-cutting machines. The research focuses on optimizing the geometric and kinematic parameters that determine machining efficiency, precision, and surface quality. Through an in-depth analysis of generating elements and surface formation models, the paper explores methods such as copying, envelopment, trace, and contact for constructing generating lines. A classification of complete and partial surface formation is provided, along with a discussion on the implications of these models for machine tool design. The results highlight the importance of rational tool path design and motion coordination to improve form-generation accuracy and tool performance. The study concludes by proposing combined formation methods to leverage the strengths of elementary approaches in practical applications.

**Keywords:** kinematic schemes, surface machining, metal-cutting machines, geometric modeling, combined methods.

## Introduction

An important stage in machine tool design is the development or selection of rational machining schemes for given surfaces, which form the basis of the machine tool's kinematics. The chosen machining scheme must ensure the required processing productivity, accuracy of surface formation, and favorable operating conditions for the cutting tool and actuating mechanisms. This is achieved, in particular, by stabilizing cutting speeds and forces, optimizing the working angles of cutting edges, reducing inertial loads, etc. The synthesis of a kinematic machining scheme includes analyzing possible surface formation methods and selecting the most rational one, designing the overall machining scheme, defining the structure and parameters of actuating movements, optimizing the kinematics of surface formation, and more. The theoretical and methodological basis for the synthesis of machining schemes lies in the general principles of the theory of surface formation by cutting. Applying these principles ensures a well-founded approach to solving the key tasks of the first stage of functional design for metal-cutting machines. Let's consider some of these principles, which are important for developing the machine tool's kinematics.

## Research Methods

Real surfaces of machine parts are an approximation of geometric (ideal) surfaces; therefore, the process of machining a real surface is based on the formation of the corresponding geometric surface. According to the well-known kinematic method of surface analysis, a geometric surface is considered as the trace formed by the motion of one line—the generatrix—along another line—the directrix [2]. These lines are called generating elements.

A trace is understood as the geometric locus of a moving generatrix line. For example, to form a plane, a straight line (Fig. 1a) or a planar curve (Fig. 1b) as the generatrix (line 1) must be moved along a straight directrix (line 2). A straight line 1, when moved along a circular directrix 2 (Fig. 1c), i.e., when rotating around axis 3, generates a cylindrical surface. The same surface can also be obtained by moving a circular generatrix 1 along a straight directrix 2 (Fig. 1d).Thus, in this case, the surface shape remains unchanged if the generatrix and the directrix are swapped.

In many cases, such a substitution is not possible. For example, to generate a conical surface (Fig. 1d), a straight generatrix 1, fixed at its apex, must move along a circular directrix 2 (i.e., rotate around axis 3). However, if this circle is moved along the straight line 1, the resulting surface will not be conical but cylindrical. Forming a conical surface with a circular generatrix is possible only if its diameter continuously changes according to a specific law during movement—in other words, if the generatrix has a variable shape. In general, both generating elements can have variable shapes, and any lines belonging to the given surface can be chosen as generatrices and directrices.

To simplify the implementation of the surface generation process, it is important to select generating lines with simple geometric shapes from the many possible options. This task is solved by analyzing the surface geometry. For example, a one-sheeted hyperboloid surface can be generated by moving a hyperbola 1 (Fig. 1e) along a circular directrix 2 (i.e., by rotating around axis 3). However, the same surface can also be formed by rotating a straight line 4, which intersects axis 3 at a certain angle. The second method is often used in practice because, in many cases, it is easier to implement and ensures higher accuracy in surface generation. Choosing a rational form for the generating lines is essential for synthesizing an optimal machining scheme.

Thus, from a kinematic perspective, surface generation is reduced to the formation of generating lines and their relative movement. The motions that ensure the formation of generating lines and their relative displacement are called form-generating motions.

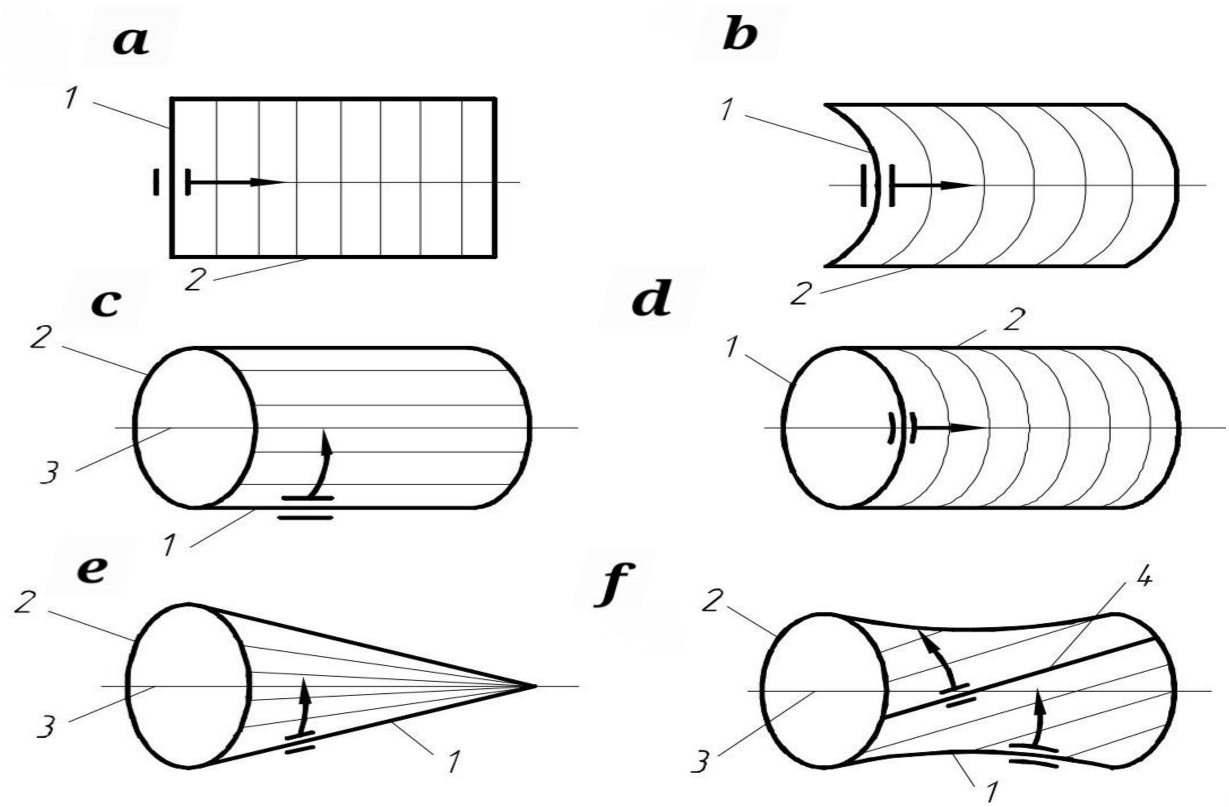


Fig. 1. Surface generation schemes: *a*, *b* — planes; *c*, *d* — cylindrical surface;
*e* — conical surface; *f* — one-sheeted hyperboloid.

Geometric Models of Surface Formation. A distinctive feature of surface generation by cutting is that the relative movement of the workpiece and the tool is accompanied by the removal of a metal layer from the workpiece. During the contact time with the workpiece, each cutting edge creates an elementary surface, whose generatrix is the generatrix of the cutting surface. The intersection of multiple surfaces formed by all cutting edges during machining constitutes the machined (real) surface, which to a certain extent corresponds to the nominal surface of the product. The generating elements of the cutting (or deforming) part of the tool come into contact with the nominal surface during machining, performing either complete or partial (incomplete) surface formation.

Complete surface formation is possible if the generating elements are mapped onto all points of the nominal surface. If this condition is not met, incomplete (partial) surface formation occurs, leading to permissible deviations of the machined surface from the nominal one.

Complete and partial formation of the machined surface can be achieved using either a single elementary surface or multiple elementary surfaces. If the machined surface is generated by a single elementary surface that is congruent with the nominal surface (e.g., in broaching or turning with a straight-edged cutting tool), complete surface formation occurs. In this case, the intersection of all points of the machined surface $M_0$ and the nominal surface $M_N$ results in the nominal surface itself: $M_0 1 M_N = M_N$. This corresponds to the first geometric model of surface formation, where the surface is considered as the trace of a moving line.

The machined surface may consist of a single elementary surface that shares a set *P* of common points with the nominal surface, forming either a surface, a line, or a collection of these elements. Mathematically, this can be expressed as: $M_0 1 M_N = P$, where *P* with $M_N$. For example, in turning with a tool having a point-shaped generating element, the elementary surface is helical and shares a set *P* of points with the nominal surface, forming a helical line. If the tool is additionally given an oscillating motion in the radial or tangential direction, the set *P* consists of discrete points or segments of the helical line. In such cases, only partial surface formation is possible.

When the machined surface is formed by multiple elementary surfaces ($M_0 = \{M_e\}$), two cases are possible. In the first case, each elementary surface $M_e$ is congruent with a section of the nominal surface ($M_e$ with $M_N$), and the machined surface is congruent with the nominal surface: $M_0$ P $M_N = M_N$. As a result, complete surface formation is achieved. An example of this is planing a surface in multiple passes using a tool with a linear generating element, where the tool's length is smaller than the workpiece width, or using multiple such tools simultaneously.

In the second case, the machined surface does not coincide with the nominal surface. Instead, the nominal surface is congruent with the envelope of one or more families of elementary surfaces: ($M_0$ P $M_N = \{P\}$, where P = $M_N$ P $M_e$. This corresponds to the second geometric model of surface formation. The contact between the elementary surface and the nominal surface, represented by the set P, typically occurs at a single point, leading to inevitable form-generation errors in the form of deviations between the machined and nominal surfaces. This model is characteristic of machining complex surfaces, particularly in cyclic (point-by-point or strip-wise) surface generation. The machined surface is formed by the intersection of multiple strips (Fig. 2), each of which may be the trace of a cutting edge, the envelope of the initial tool surface in its relative motion, or a set of auxiliary surfaces.

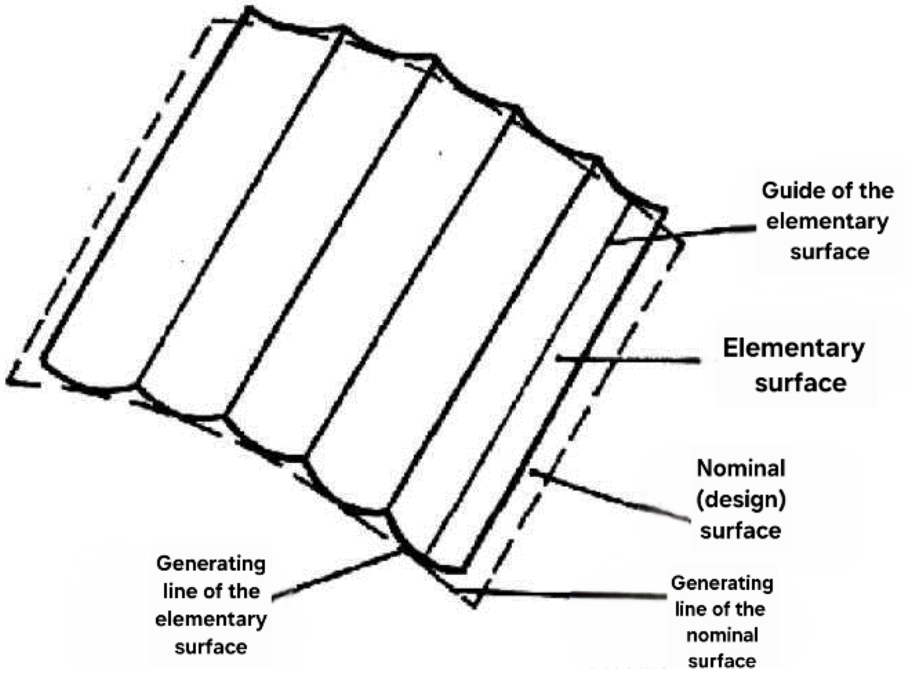

Fig. 2. Scheme of Incomplete Surface Formation for a Complex Surface.

If each strip is considered an elementary surface, then the nominal surface is the envelope of at least a two-parameter family of elementary surfaces. The geometric parameters of these surfaces affect Form-generation accuracy, Shape and working conditions of the cutting tool, Kinematics of surface formation. This highlights the need to select a rational form of elementary surfaces when developing a machining scheme.

A common feature of the first and second models of surface formation is that the surface is generated by the relative motion of the generating element with respect to the workpiece. In the first model, the machined surface is the trace of the generatrix. In the second model, the machined surface is the envelope of the initial tool surface. An important condition for the feasibility of surface formation in the second model is ensuring tool passability, meaning that the initial tool surface must not intersect the nominal surface of the workpiece [3].

The shape of the trace of the generating line or the envelope of the generating surface in a section perpendicular to the direction of relative motion is called the characteristic image of the tool [2]. In complete surface formation schemes, this characteristic image is congruent with one of the possible generatrices of the machined surface. The characteristic image expresses the common geometric features of possible tools that are essential for the surface formation process. Additionally, its shape, position, and direction of movement in the coordinate system associated with the workpiece are the defining features of the overall machining scheme.

If the characteristic image of the tool is taken as the generatrix of the surface, then the surface formation process can be considered as the relative movement of this image [2]. This model of surface formation, considering that the characteristic image of the tool can continuously change during movement, is universal.

The requirement for the characteristic image of the tool to change shape during its relative motion may be due, in particular, to the need to approximate the shape of the generatrix of the nominal surface of the workpiece. The possibility of such a change must be ensured during the design of the machine tool or cutting tool.

The degree of approximation of the machined surface to the nominal surface depends on the correspondence between the shape of the characteristic image of the tool, the trajectory of its movement, and the generating lines of the nominal surface. Therefore, when synthesizing a surface formation scheme, selecting a rational shape for the characteristic image is crucial. For a complex surface, multiple solutions to this problem are possible. For example, a convex surface can be formed using a tool with a convex, straight, or concave characteristic image, depending on the direction of its movement. Thus, for the synthesis of rational surface formation schemes, it is important to evaluate the impact of the overall machining scheme on efficiency parameters, using objective criteria to select the shape of the characteristic image of the tool and the trajectory of its relative movement.

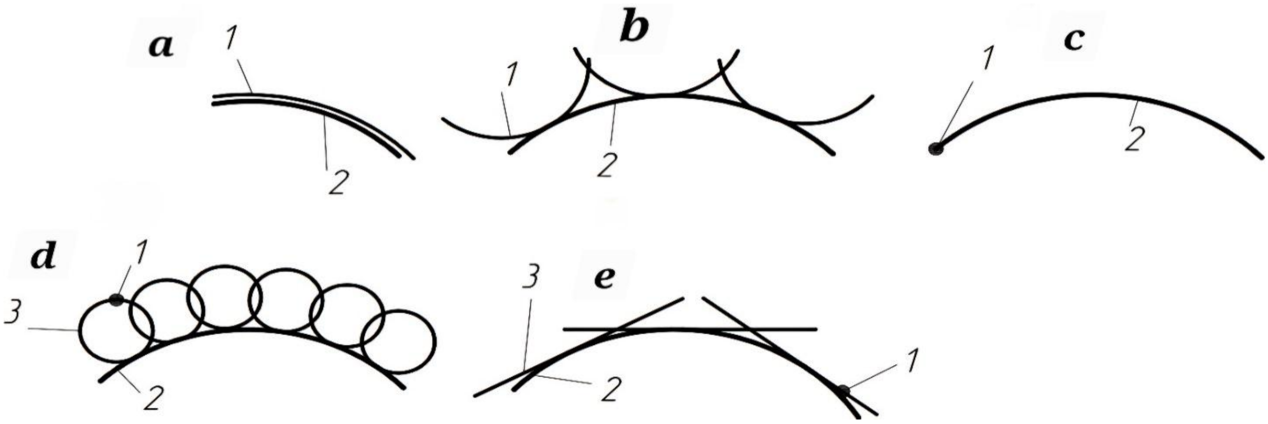


Fig. 3. Methods of generating line formation: a – copying; b – envelopment; c – trace; d, e – contact.

Methods of Generating Lines Formation. When machining a surface on a machine tool, its generating lines must be reproduced through the movements of the tool and the workpiece and by the forming (generating) elements of the tool that come into contact with this surface. These forming elements can be either lines (cutting edges) or points (apexes of cutting blades). In the first case (when forming elements are lines), generating lines are formed using the copying method (Cp) or the envelopment method (En). In the second case (when forming elements are points), generating lines are formed using the trace method (Tr) or the contact method (Cn) [4]. Let's examine the implementation of these methods on machine tools.

Copying Method. In the copying method, the generating element 1 (Fig. 3, a) of the tool matches the shape and extent of the generated line 2. As a result, no formation movement is required—only a movement to transfer the generating element from its initial to its final position.

In this case, the tool itself serves as the physical carrier of the generated line's shape, making this method a geometric method of line formation.

The advantage of the copying method is the simple kinematics of the machine. However, its disadvantage lies in the complexity and lack of versatility of the tool when machining profiled surfaces. This method is widely used in practice for profiling internal and external surfaces of both simple and complex geometries using blade and abrasive tools (such as broaching, form turning, milling, etc.).

The rolling method forms line 2 (Fig. 3, b) as the envelope of multiple successive positions of the generating element 1 during its relative motion. This requires a single complex shaping movement. In this method, both the tool and the machine's kinematics serve as the material carriers of the generated line's shape, creating a rolling motion, which can be either continuous or intermittent. This method is widely used for machining complex surfaces, such as gear wheels, to profile the cut teeth. The tool's generating elements are shaped according to the tooth profile of the part, which may engage with the machined gear. The machine's kinematics reproduces a specific machine gear engagement (such as cylindrical, rack-and-pinion, or worm gear engagement).

In the trace method (Fig. 3, c), the generated line 2 represents the trajectory of the generating element, which is a point 1, such as the tip of a lathe tool. This means that only one shaping movement is required. This method is classified as kinematic because the machine's kinematics is the only material carrier of the generated line's shape. The trace method is highly versatile in forming various line shapes, making it widely used for machining surfaces of both simple and complex geometries.

In the contact method (Fig. 3, d), the generated line 2 is formed kinematically as a tangent to a set of auxiliary lines 3, which are the trajectories of the generating point 1. This method requires two shaping movements: One movement to create the auxiliary lines.

Another movement for their relative displacement. The machine's kinematics serves as the material carrier of the generated line. The contact method is typical for machining with rotating tools (e.g., milling cutters, grinding wheels). In such cases, the auxiliary lines are usually circles, but they can also be straight lines (Fig. 3, e) or take other forms.

The difference between the contact method and the rolling method lies in how the auxiliary lines are formed: In the contact method, the auxiliary lines are generated kinematically, usually by the trace method and less often by the contact method itself. In the rolling method, the auxiliary lines are determined by the shape of the generating elements of the tool.

The four considered methods of generating generating lines are elementary (basic) methods. However, combined methods are also possible, which represent a combination of elementary methods, such as (Cp + Tr), (En + Tr), etc. Combined methods allow combining the advantages and avoiding the disadvantages of the elementary methods included in them.

For example, forming a generating line using the copying method during cylindrical turning by plunge cutting eliminates the possibility of surface roughness in the form of ridges, which are inevitable when turning with a standard tool due to the point contact of its cutting edge with the nominal cylindrical surface of the workpiece. However, in the copying method, as the length of contact between the tool's cutting edge and the workpiece increases, undesirable vibrations may occur. These vibrations are usually not present when forming the generating line of a cylindrical surface using the trace method.

**Results and discussions**

Forming this generating line using a combined method (Cp + Tr) allows for combining the advantages of the basic methods—eliminating the possibility of ridge formation and preventing excessive vibrations during cutting. This method is implemented in turning with a tool that has a straight transitional cutting edge, which is parallel to the feed direction and longer than the feed per revolution of the workpiece (turning with Kolesov tools).

Surface Formation. The generating and guiding elements of a given surface can be obtained using any of the methods discussed earlier. Therefore, surface formation methods are determined by possible combinations of methods for generating lines. Since the speed of generating formation cannot be lower than that of the guiding element, the following surface formation methods—based on elementary methods of generating line formation—are possible: Cp – Cp, Cp – Tr, Cp – En, Cp – Cn, En – En, En – Tr, Tr – En, Tr – Tr, Tr – Cn, Cn – En, Cn – Tr, Cn – Cn.

There is a certain relationship between temporal and geometric formation schemes, but in many cases, there is no one-to-one correspondence. Therefore, it is advisable to indicate the temporal characteristics of the formation process when designating geometric schemes, which is important for designing the kinematics of the machine tool. For example: Obn – denotes a continuous process of line formation by the enveloping method (Ob), Obp – denotes an intermittent process of line formation by the enveloping method (Ob). These designations are used in various gear shaping machines with different kinematics.

## Conclusion

The synthesis of kinematic schemes for surface machining plays a crucial role in the functional design of metal-cutting machines. By analyzing the geometric models of surface formation and the principles of generating line construction, this study demonstrates how tool motion, surface geometry, and tool shape interact to determine the efficiency and accuracy of the machining process. The classification of formation methods—copying, envelopment, trace, and contact—provides a foundation for selecting optimal machining strategies based on surface complexity and tool geometry. Furthermore, the use of combined methods enables the integration of the advantages of elementary approaches, enhancing process stability and surface quality. Incorporating both temporal and geometric aspects into kinematic scheme design ensures a comprehensive and rational approach to machine tool development. Future research should focus on further refining these schemes through simulation and practical validation to increase their adaptability in advanced manufacturing environments.

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